Department of Basic Science Level: 1 Examiner: Dr. Mohamed Eid Time allowed: 3 hours	Pyramids higher Institute P.H.I. For Engineering And Technology معهد الأهرامات العالى للهندسة و التكنولوجيا	Prep. Year: Final Exam Course: Mathematics 1 Course Code: BAS 013 A Date: May , 2016
Answer all questions	No. of questions: 5	Total Mark: 70

Question 1 Find **y**\` from the following: (a) $y = 4x^4 - \cos x$ (b) $y = x^3 . \tan x$ (c) $y = \sin x + sin^3 x$ (d) $y = (x^3 + 3)^5$ (e) $y = \frac{2}{3} + \frac{\cos x}{x^4}$ (f) $\mathrm{y} = \frac{\mathrm{x}}{3} + \frac{3}{\mathrm{x}}$	18
Question 2 Find the limits: (a) $\lim_{x\to 1} \frac{\sqrt{x} - 1}{1 - x^4}$ (b) $\lim_{x\to 0} \frac{x^2 - \tan x}{x + \sin 2x}$ (c) $\lim_{x\to 0} \frac{\sin x}{x^2 + 3x}$ (d) $\lim_{x\to\infty} \frac{x + x^5}{4 + 2x^5}$	8
Question 3 (a)Find the extrema of each function : (i) $f(x) = 1 + 12x - x^3$ (ii) $g(x) = 2 + x^3$	10
(b) Write the Maclurin's expansion of the function : $f(x) = x . \cos x .$	4
Question 4 (a)State the definition of the circle.	3
(b)Write the equation of circle where the points (2, 3), (1, –2) are ends of diameter.	4
(c)Show that the following circles are orthogonal and find the radical axis : $x^2 + y^2 + 2x + 4y - 1 = 0$, $x^2 + y^2 - 4x + 3y + 3 = 0$. Also, find the points of intersection.	8
Question 5 (a) State the definition of the parabola.	2

(b)Find the angle between the lines: $x^2 + xy - 2y^2 = 0$ and separate them.	4
(c)Write the equation of line which passes through the points : (2, 3), (1, –2).	4
(d)Find center, vertices and sketch the parabola : $y^2 - 8x + 6y + 33 = 0$.	5

Good Luck *Dr. Mohamed Eid*

Answer

Question 1

(a) $y` = 16x^3 + \sin x$ (b) $y` = 3x^3 . \tan x + x^3 . sec^2 x$

(c) $y` = \cos x + 3sin^2 x \cos x$ (d) $y` = 5(x^3 + 3)^4 (3x^2)$

(e) $y` = -\sin x . x^{-4} - 4\cos x . x^{-5}$ (f) $y` = \frac{1}{3} - \frac{3}{x^2}$

--18-marks

Question 2

(a) $\lim\limits_{x\to 1} \frac{\sqrt{x} - 1}{1 - x^4} = \frac{0}{0} = -\frac{1}{8}$

(b) $\lim\limits_{x\to 0} \frac{x^2 - \tan x}{x + \sin 2x} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{2x - \sec^2 x}{1 + 2\cos 2x} = \frac{0 - 1}{1 + 2} = -\frac{1}{3}$

(c) $\lim\limits_{x\to 0} \frac{\sin x}{x^2 + 3x} = \frac{0}{0} =$ (c) $\lim\limits_{x\to 0} \frac{\cos x}{2x + 3} = \frac{1}{0 + 3} = \frac{1}{3}$

(d) $\lim\limits_{x\to \infty} \frac{x + x^5}{4 + 2x^5} = \frac{\infty}{\infty} = \lim\limits_{x\to \infty} \frac{\frac{1}{x^4} + 1}{\frac{4}{x^5} + 2} = \frac{0 + 1}{0 + 2} = \frac{1}{2}$

--8-marks

Question 3

(a)(i) $f`(x) = 12 - 3x^2 = 0$. Then $x^2 = 4$. Then $x = \pm 2$.

Since $f``(x) = -6x$.

Then $f``(2) = -12$, then 2 is maximum.

Then $f``(-2) = 12$, then – 2 is minimum.

--6-marks

(ii) $g`(x) = 3x^2 = 0$. Then $x = 0$. $g``(0) = 6(0) = 0$, it fails.

By the first derivative test.

$g`(-1)$	x = 0	$g`(1)$
3	Neither nor	3

--4-marks

(b) Since $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ ....

Then $f(x) = x\cos x = x - \frac{x^3}{2!} + \frac{x^5}{4!}$ ....

--4-marks

Question 4

(a)Definition of the circle.

--3-marks

(b) $(x-2)(x-1) + (y-3)(y+2) = 0$

Or $\mathrm{x}^2 + \mathrm{y}^2 - 3\mathrm{x} - \mathrm{y} - 4 = 0$

--4-marks

(c)Since $\mathrm{g}_1\mathrm{g}_2 + \mathrm{f}_1\mathrm{f}_2 = -2 + 3 = 1 = \frac{1}{2}(\mathrm{c}_1 + \mathrm{c}_2)$

Then, the circles are orthogonal.

The radical axis is : 6x + y – 4 = 0 Or y = 4 – 6x.

Solving the radical axis with the first circle, we get

$$x^2 + (4 - 6x)^2 + 2x + 4(4 - 6x) - 1 = 0,$$

Then $37x^2 - 70x + 31 = 0$ and $x = 1.18,\ \ x = 0.71$

Then, the points of intersection are : (1.18, – 3.1), (0.71, – 0.26)
--8-marks

Question 5

(a)Definition of the parabola.
--2-marks

(b)The angle between the lines is: $\tan\theta = \pm 2\frac{\sqrt{\frac{1}{4}+2}}{1-2} = \pm 3$

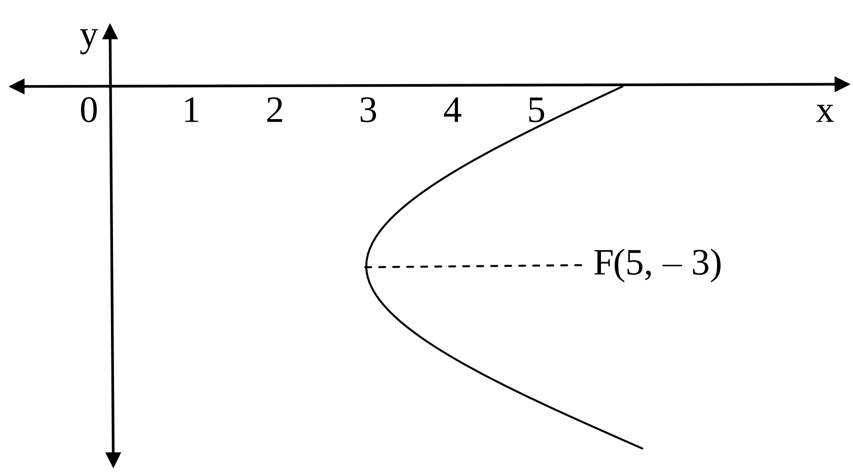
The lines are : $(x + 2y)(x - y) = 0 \quad Or \quad x + 2y = 0, \quad x - y = 0$
--4-marks

(c)The equation of line is : $\frac{y-3}{x-2} = \frac{-5}{-1}$ Or 5x – y – 7 = 0.

--4-marks

(d)From $y^2 - 8x + 6y + 33 = 0$ Or $(y + 3)^2 = 8(x - 3)$.

The vertix is : (3, – 3), a = 2, focus (5, – 3).



--5-marks

Quiz 1-Math 1

[1]Find **y`** from the following:

(a) $y = \frac{x^3}{3} - 3\cot 2x$

(b) $y = 3x^{-4} + \sin x^2$

(c) $y = (2x + \cos x)^3$

(d) $y = \tan x^3 + \cos^3 x$

(e) $y = \sqrt{3} + \sec(2x + 1)$

[2]Find the limits:

(a) $\lim\limits_{x\to 2} \frac{x^3 - 8}{\sqrt{x} - 1}$

(b) $\lim\limits_{x\to 0} \frac{\tan 2x}{x + x^2}$

(c) $\lim\limits_{x\to \infty} \frac{3 + 2x^3}{x^2 + 2x}$

[3](a) Determine the extrema and inflection points of the function:

$f(x) = x^3 - 3x^2 - 9x.$

(b)Write the Maclurin's expansion of the function:

$f(x) = x^2 + \sin x.$

Quiz 2-Math 1

[1]State the definition of parabola.

[2]Show that the circles:

$x^2 + y^2 - 2x + 4y = 0, \quad x^2 + y^2 + 2x + 2y + 2 = 0$

are orthogonal and find the radical axis.

[3]Write the equation of circle where the points:

(2, 0), (1, –3) are ends of its diameter.

[4]Find the vertex, focus and sketch the parabola:

$x^2 - 2x + 4y + 5 = 0.$

Math. 1 Mid-Term Exam – Spring 20 Time: 1 Hour

[1]Find **y`** from the following:

(a) $y = 3x^4 - 2\cos x$ (b) $y = x^{-3} + 3\sin 2x$

(c) $y = (x + \sec x)^5$ (d) $\mathrm{y} = \sin \mathrm{x}^2 + \sin^2 \mathrm{x}$

(e) $y = \frac{\tan\ x}{x^5}$

[2]Find the limits:

(a) $\lim\limits_{x\to 1} \dfrac{x^5 - 1}{x^7 - 1}$ (b) $\lim\limits_{x\to 0} \dfrac{x - \sin 2x}{x + 2\tan x}$ (c) $\lim\limits_{x\to\infty} \dfrac{x^3 - 2x}{1 + x^2 + x^3}$

[3](a)Determine maximum and minimum points of the function:

$$f(x) = x^3 - 3x + 1.$$

(b)Write the Maclurin’s expansion of the function:

$$f(x) = x^2 + \frac{1}{x+2}.$$